Using chi-square and T-square charts to interpret sieve analysis data

After a material's particle size distribution is determined, generally by sieve analysis, statistical process control procedures can be used to interpret the results. However, these procedures produce a cumbersome amount of data and aren't always accurate.

This article describes two types of statistical process control charts — chi-square and T-square charts — that you can use to simply and effectively interpret sieve analysis results. The information on using chi-square and T-square charts can also be applied to results generated by particle sizing methods other than sieve analysis.

Sieve analysis and statistical process control procedures that interpret sieve analysis data are commonly used to determine changes in particle size distribution. However, using these techniques involves several steps, requires many sieves, and produces a large amount of data.

A sieve analysis determines the particle size distribution of a material fraction — that portion of raw material that falls within a certain particle size range. The fraction is typically referred to by its particle size range: for example, a 3 x 10 fraction contains particles between 3 and 10 mesh.

In a sieve analysis, several fractions from many batches (for example, batches from different railcar loads) are tested. A sample is taken from each batch and run through a series of sieves with different mesh sizes. After the sample is sieved, a part of it is retained on each sieve. The retained material is weighed, and its percentage by weight of the entire sample is determined. Because samples from many different batches are tested using several sieves with different mesh sizes, and because several fractions may be tested, a large amount of data is generated.

Statistical process control procedures, including variable control charts such as X charts and moving range (or sigma) charts, are then used to monitor the particle size distribution of the different batches. Standard variable control charts, which typically present data for one sieve size across many batches of a fraction, can help you determine if the mean and standard deviation of the data for one or more sieve sizes have changed, thus indicating changes in the particle size distribution.

However, standard variable control charts have several drawbacks. First, the large number of sieves used in each sieve analysis, as well as the large number of batches and fractions tested, requires making and interpreting many charts. Second, because many tests are performed and many charts are made for each material fraction, the chances of making errors increase; for example, the results may show that the particle size distribution of a given fraction has changed when, in fact, it hasn't. Third, the results don't always show whether the particle size distribution has changed from the distribution the final product requires.

This article describes two types of statistical process control charts — chi-square and T-square charts — that you can use to overcome the drawbacks of standard variable control charts. The charts use the entire shape of a particle size distribution, rather than just the mean and standard deviation of several sieves. The chi-square and T-square charts enable you to determine changes in the particle size distribution of a fraction by using one number (either the chi-square or T-square value), thus requiring only one chart to present all the data for each material fraction.

Using standard variable control charts

To understand how chi-square and T-square charts are used, it may be helpful to look first at how a standard statistical process control procedure uses histograms and variable control charts.

A typical histogram (Fig. 1a) takes the form of a relative frequency distribution and graphically presents the sieve analysis data. The histogram plots the percentage of the total sample by weight versus the percentage of the sample passing a sieve of a particular mesh size. A standard variable control chart can be generated for each mesh size to show any change in the percentage of material remaining on a particular sieve over a series of samples.

However, using a standard variable control chart to monitor changes in particle size distribution can be difficult and isn't always accurate. For example, consider three sieve analysis histograms for three batches of the same material fraction: batch 1 (Fig. 1a), batch 2 (Fig. 1b), and batch 3 (Fig. 1c). If you used a standard variable control chart to interpret the 20- and 28-mesh sieve data for these histograms (and others in the series), it would require two separate charts, one for the 20-mesh sieve (Fig. 2a) and one for the 28-mesh sieve (Fig. 2b). The charts would probably help you detect a shift between batch 1 and batch 2 because the mean of the percentages retained on the two sieve sizes has changed significantly from batch 1 to batch 2.

However, standard variable control charts would probably not help you detect a process shift between batch 2 and batch 3 for the same sieve sizes. In this case, the mean of the percentages of particles retained on the 20- and 28-mesh sieves for each batch is similar, even though the shape of the batch 3 distribution is skewed in the opposite direction from that of batch 2, indicating a major
Fig. 1  Histograms for sieve analyses of three batches of material

a. Batch 1

b. Batch 2

c. Batch 3
Fig. 2 X chart for Fig. 1 data

a. 20 mesh

Upper control limit: 27.20
Mean: 20.92

b. 28 mesh

Upper control limit: 28.37
Mean: 21.61
Using the chi-square chart

Chi square is a process control value that can be used to interpret sieve analysis data. In essence, the chi-square value represents — in one number — the disparity between historical sieve analysis data and new sieve analysis data. Thus, the chi-square value enables you to create one chart — called the chi-square chart¹ — that represents data for several sieve sizes and many batches and monitors the entire shape of the particle size distribution of a material fraction.

Defining the chi-square value. The concept of chi square can be explained by looking at Table I. The table shows how the chi-square value is defined for a sieve analysis using five sieve sizes. It compares historical sieve analyses with a new sieve analysis of the same fraction of raw material.

The second column shows the average number of particles retained on five sieve sizes for previous sieve analyses of batches from the fraction of raw material. The third column shows the number of particles retained on the same sieve sizes for a new batch. The fourth column lists the difference between the historical average number of particles retained and the new batch’s number of particles retained for each sieve size; the fifth column lists the square of each difference. The last column lists the square of the difference divided by the historical average number of particles for each of the sieve sizes. The total of the values in the last column is the chi-square value, which represents the disparity of the new batch’s number of particles with the historical average number of particles.

A large chi-square value, which represents a large degree of disparity between the numbers of particles retained in the historical batches and the new batch, indicates the process has shifted; a small chi-square value, which represents a small degree of disparity, indicates that the process has remained relatively consistent.

The 99.5 percent chi-square value (based on standard chi-square table values)¹ is 14.86 for 4 degrees of freedom (the number of degrees of freedom is derived from the number of sieve sizes used minus 1 — in Table I’s case, 5 - 1 = 4). This means that 99.5 percent of the chi-square values for analyses using five sieve sizes are less than 14.86. In Table I, the chi-square value is 154.07. Thus, you can conclude that this chi-square value, which shows a large disparity between the historical average and the new batch, is too large to be a chance event, and its cause should be investigated.

Converting sieve analysis weight data to the number of particles. One problem with using the chi-square value is that sieve analysis reveals the weight of the particles retained on each sieve, rather than the number of particles. While you might be able to count the number of particles retained on large sieve sizes, it would be impossible to count the particles retained on small sieve sizes.

Converting the weight data to the number of particles requires two steps. First, estimate the number of particles retained on each sieve. To do this, take a sample of material from each sieve to determine the average weight of the particles; then divide the weight of the particles retained on each sieve by the average weight per particle, which produces the estimated number of particles retained on each sieve.

Second, adjust the data so the average value of chi square is the known average, which is the number of degrees of freedom (again, the number of sieve sizes used minus 1). To do this, calculate the chi-square values for each batch and determine the average of all the chi-square values. Then, multiply the estimated number of particles retained on each sieve by the number of degrees of freedom (average value of chi square).³

Plotting the chi-square chart. You can use the chi-square value to generate a chi-square chart by plotting the chi-square value of the sieve analyses for each batch tested. Because the chi-square value condenses historical and new data for several sieve sizes into one number and allows several batches to be plotted in one chart, the chi-square chart monitors the shape of a material fraction’s entire particle size distribution. This provides an instant comparison of the particle size distribution of the historical average and new batch and makes it much easier to detect a process shift.

Comparing the chi-square chart to a standard variable control chart. Now compare a standard variable control chart, the X chart (Fig. 3), with a chi-square chart (Fig. 4). Both charts present sieve analysis data for 32 batches of a 3 x 6 material fraction. Figure 3 plots the percentage of the weight of the total sample retained on one sieve — 4 mesh — for 32 batches. The chart’s upper control limit (56.33) and lower control limit (36.53) are based on the data’s

Table I  Chi-square table for a sample sieve analysis using five sieve sizes

<table>
<thead>
<tr>
<th>Sieve size (mesh)</th>
<th>Historical average number of particles</th>
<th>New batch’s number of particles</th>
<th>(Difference)²</th>
<th>Historical average number of particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>500</td>
<td>425</td>
<td>- 75</td>
<td>5,625</td>
</tr>
<tr>
<td>4</td>
<td>700</td>
<td>720</td>
<td>+ 20</td>
<td>400</td>
</tr>
<tr>
<td>6</td>
<td>400</td>
<td>280</td>
<td>-120</td>
<td>14,400</td>
</tr>
<tr>
<td>8</td>
<td>200</td>
<td>300</td>
<td>+ 100</td>
<td>10,000</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>175</td>
<td>+ 75</td>
<td>5,625</td>
</tr>
</tbody>
</table>

Chi square 154.07

¹ Chi square

² (Difference)²

³ Average

Chi square

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moving range. The weight on the 4-mesh sieve varies randomly from batch to batch within these limits, thus showing no clear shift in the process at any particular point.

Figure 4 plots the chi-square value for five sieves — 3, 4, 6, 8, and 10 mesh — for 32 batches. The chart’s upper control limit is 12.49 and its mean is 4.00. By presenting sieve analysis data for more than one sieve size in one plot, as well as taking into account historical sieve analysis data for the same sieve sizes, the chi-square chart clearly shows a radical process shift at batch 28, where the line rises sharply over the upper control limit.

**Using the T-square chart**

*T square*, known as Hotelling’s T square, is another value that can be used to interpret sieve analysis data. Using the T-square value to plot a T-square chart with sieve analysis data can help you simultaneously analyze the data for changes in means and changes in relationships between results for particular sieve sizes. Thus, a T-square chart will clearly show any shift in the particle size distribution. Unlike a chi-square value, a T-square value doesn’t require that weight data be converted to the number of particles before the chart is plotted.

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**Fig. 3** X chart for a 32-batch sieve analysis showing the percentage (by weight) of the total sample retained on a 4-mesh sieve

**Fig. 4** Chi-square chart for a 32-batch sieve analysis showing the value of chi-square
The value of $T$ square is expressed in matrix terms as:

$$N \times (\bar{x} - \mu) \times D^{-1} \times (\bar{x} - \mu)$$

where:

- $N =$ Batch size
- $D^{-1} =$ Inverse of the variance/covariance matrix
- $\bar{x} =$ Vector of average values on each sieve in a batch
- $\mu =$ Vector of average values over all batches

The $T$-square value is a generalization of the Student's $t$ statistic, which, in the one-variable case, is defined as:

$$t = \frac{\bar{x} - \mu}{S/\sqrt{N}}$$

and thus:

$$t^2 = \frac{N (\bar{x} - \mu) (\bar{x} - \mu)}{S \times S}$$

where:

- $\bar{x} =$ Sample average
- $\mu =$ Hypothetical average
- $N =$ Sample size
- $S =$ Standard deviation

which is the same form as Hotelling's $T$-square.

Figure 5 is a $T$-square chart that plots the $T$-square value for the same five sieve sizes and same 32 batches of a $3 \times 6$ material fraction presented in Figures 3 and 4. The upper control limit is 14.43 and the mean is 5.71. Because the $T$-square chart, like the chi-square chart, compares the data for several sieve sizes at once, Figure 5 clearly shows a process shift at batch 28, where the plot line rises sharply above the upper control limit.

**Conclusion**

Plotting a chi-square or $T$-square chart for sieve analysis data is an effective statistical process control method for monitoring and controlling the particle size distribution of your raw material. Both charts enable you to check the entire shape of the particle size distribution with a single number — either the chi-square or $T$-square value. As a result, you only need to create one chart, rather than several, for each fraction of raw material you want to control.

**References**


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